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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2215)

**PEM0026 – TRIGONOMETRY & GEOMETRY**  
(Foundation in Engineering)

17 FEBRUARY 2023  
9.00 a.m – 11.00 a.m  
(2 Hours)

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### INSTRUCTIONS TO STUDENT

1. This question paper consists of **SIX (6)** printed pages including the cover page and appendix.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.

**APPENDIX****Formula****Trigonometry Identities**

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

$$\sin(\theta) = \cos(90^\circ - \theta)$$

$$\csc(\theta) = \sec(90^\circ - \theta)$$

$$\tan(\theta) = \cot(90^\circ - \theta)$$

$$\cos(\theta) = \sin(90^\circ - \theta)$$

$$\sec(\theta) = \csc(90^\circ - \theta)$$

$$\cot(\theta) = \tan(90^\circ - \theta)$$

$$\cos^2 A + \sin^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\csc \theta = \sec(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta)$$

$$\cot \theta = \tan(90^\circ - \theta)$$

$$z_k = \sqrt[n]{r} \left[ \cos \left( \frac{\theta}{n} + \frac{360^\circ k}{n} \right) + i \sin \left( \frac{\theta}{n} + \frac{360^\circ k}{n} \right) \right], \text{ where } k = 0, 1, 2, \dots, n-1$$

**QUESTION 1 [25 marks]**

(a) Find the exact value of each the expression. Do not use calculator.

(i)  $\sin^2 20^\circ + \csc^2 45^\circ - \cos^2 70^\circ$ . (2 marks)

(ii)  $1 - \tan^2 \frac{\pi}{12} + \sec^2 \frac{\pi}{12}$ . (2 marks)

(iii)  $4 \sin \frac{4\pi}{3} + 5 \cos \frac{5\pi}{6}$ . (3 marks)

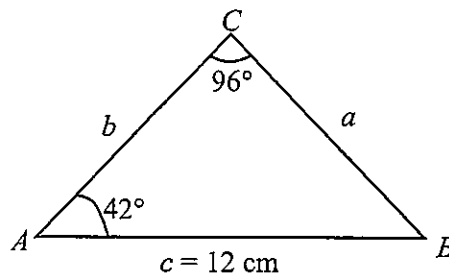
(b) Given the function:  $y = -2 \sin \left( \frac{3}{2}x - \pi \right) + 2$ ,  $\frac{2\pi}{3} \leq x \leq \frac{10\pi}{3}$ .

(i) Determine the amplitude, period and phase shift of the function. (5 marks)

(ii) Draw the graph of the above function for 2 cycles. (6 marks)

(c) If  $\sin \alpha = -\frac{3}{5}$ ,  $\cos \alpha < 0$ , then find the exact value of  $\cot \alpha$ . (3 marks)

(d) Solve the triangle below. Round the lengths of sides to the nearest tenth and angle measures to the nearest degree. (4 marks)



**Continued...**

**QUESTION 2 [25 marks]**

- (a) Use a sketch to find the exact value of  $\csc \left[ \cos^{-1} \left( -\frac{\sqrt{3}}{2} \right) \right]$ . Do not use a calculator. (4 marks)

- (b) Prove the identity:  $\frac{\sin \theta}{-1 - \cot \theta} + \frac{\cos \theta}{1 + \tan \theta} = \cos \theta - \sin \theta$ . (7 marks)

- (c) Given that  $\sin x = \frac{1}{3}$  and  $\sin y = \frac{2}{5}$ ,  $x$  in Quadrant II and  $y$  in Quadrant I.

(i) Find the value of  $\tan (x - y)$ . (5 marks)

(ii) Find the exact value of  $\cos 2y$ . (3 marks)

- (d) Solve the trigonometric equation:

$$\cos 2x = 6 \cos^2 x - 2 \quad \text{where } 0^\circ \leq x \leq 360^\circ \quad (6 \text{ marks})$$

**Continued...**

**QUESTION 3 [25 marks]**

- (a) Express  $\frac{(-10 + \sqrt{-125})^2}{(2\sqrt{-1})^3}$  in  $(a + bi)$  form. (5 marks)
- (b) If given  $z = 2(\cos 30^\circ + i \sin 30^\circ)$ ,  $w = 6(\cos 15^\circ + i \sin 15^\circ)$  and  $v = -5 + 4i$ .
- (i) Find  $v$  in polar form. Leave your answer in 2 decimal places. (3 marks)
- (ii) Find  $\frac{v}{wz^3}$  in rectangular form. (7 marks)
- (iii) Determine the square root of  $z$ . (4 marks)
- (c) The following vectors are given  
 $\mathbf{a} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$        $\mathbf{b} = (4p - 1)\mathbf{i} + (p - 2)\mathbf{j} + \mathbf{k}$  ; where  $p$  is scalar constant.  
Find the value of  $p$  if vector  $\mathbf{a}$  is orthogonal to vector  $\mathbf{b}$ . (3 marks)
- (d) Find the angle between vectors  $\mathbf{v} = -8\mathbf{i} + 9\mathbf{j} - 3\mathbf{k}$  and vector  $\mathbf{w} = \mathbf{i} - 2\mathbf{j} - 17\mathbf{k}$ . (3 marks)

**Continued...**

**QUESTION 4 [25 marks]**

- (a) Find an equation of a straight line that is perpendicular to the line  $3x - y + 4 = 0$  and passing through the point  $(-2, 4)$ . (3 marks)
- (b) Find the vertex, focus and directrix of the parabola  $(x + 2)^2 = -8(y + 2)$ . (4 marks)
- (c) Identify the coordinates of the center, vertices, foci and write the equations of the asymptotes of the hyperbola with an equation of  $\frac{(x+4)^2}{9} - \frac{(y+3)^2}{16} = 1$ . (9 marks)
- (d) Given that the equation of an ellipse is  $4x^2 + 9y^2 - 32x + 36y + 64 = 0$ , find the center, foci and vertices of the ellipse. Then, graph the ellipse. (9 marks)

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